THE SENIOR COLLEGE MESSENGER

Issue 15: January, 2023

This is an organ for members of Senior College to submit short articles that share news, letters to the editor, reactions to the program and anything that they feel will be of general interest. Its regular appearance will allow for an exchange of opinion of topics of interest to the members. In particular, it would be interesting to record reactions to the talks, colloquium topics and books discussed.

Please submit contributions to the editor, Ed Barbeau at barbeau@math.utoronto.ca
.

SENIOR COLLEGE CENTRE

A resource for all members of Senior College

The Senior College Centre is "the branch of Senior College that provides services and organizes activities for all College members" (Strategic Plan for the Senior College Centre, 2018). The Senior College Centre is the child of the merger between the Academic Retiree Centre with Senior College that resulted from the provostial review of the former in 2013. (For a description of the events that led to the formation of Senior College and the Senior College Centre, see *The RALUT Story* by Peter Russell.) According to the Strategic Plan, the purposes of the Senior College Centre are to "(1) to provide services of interest to academic retirees at the University of Toronto, (ii) to strengthen and maintain connections between the Centre and the University's academic retiree community, and (iii) to make the College's scholarly resources available to groups in the community outside the University."

The Centre is physically located at 256 McCaul Street where there are two meeting rooms, a small lounge provided with reading material consisting of books and other publications donated by members of the College as well as with coffee and tea, and the office of the College Administrator. During "normal" times (in the absence of Covid), the Centre is open on weekdays from 9 am to 4 pm during which hours the Administrator and/or volunteers are present to welcome visitors and to answer questions.

The Centre currently oversees, or plans to develop, a number of services and activities available to all members of the College. These include such things as the Speakers Bureau which provides speakers for groups in the community, the offering of health education courses dealing with issues relevant to seniors, providing "technology" workshops, mounting art shows of work by members of the College.

The activities of the Senior College Centre, previously overseen by the Board of Management, now fall under the mandates of the Executive Committee of Council and the new standing committee of Council, the Senior College Centre Committee. (*Jim Gurd*)

HONOURS

Peter Russell was promoted to **Companion of the Order of Canada** on June 29, 2022. He was awarded the status as Officer of the Order on December 29, 1986.

LINKS FOR THE 17th SYMPOSIUM

Recordings of the talks are now available on *YouTube* under the title "Playlist: 17th Annual Senior College Symposium": SC Symposium talks.

The cochairs of the Symposium Committee would be pleased to receive suggestions for future symposia. Their coordinates are margaret.procter@utoronto.ca and williamlogan@rogers.com.

CALENDAR OF COMING EVENTS

Events marked with **F** are for fellows and external fellows. Registration a few days ahead is necessary for each event. This can be done in response to a weekly email from Senior College to its members that describes the events or by going on line at www.seniorcollege.utoronto.ca .

Talks: Wednesdays, 2 pm

Talks will take place at the Faculty Club **and** on Zoom.

January 4: Burton Lim, *Great whales*

January 11: Sandra Rehan, *Diversity, decline and sustainability*

January 18: Aisha Ahmad, *Why jihadists win*

January 25: Paul Martin, Peter, Hajnal, John Kirton (panel), *The G20 today*
Moderator: Louis Pauly

February 1: Sophia Moreau, *Tort law and social equity*

February 8: Peter Sloly, *Reflections and projections on Canadian Policing*

February 15: Miriam Diamond, *Solutions to address the global threat of chemical pollution*

February 22: Michael Gervers, *Ethiopian Dark Ages (7th - 12th centuries)*

March 1: Bence Viola, *Neanderthals and Denisovans in Central Asia*

Book Club: Mondays 2-4 pm (Zoom only) (**F**)

January 9: Maggie O'Farrell, *Hamnet* (Leader: Molly Wills)

February 6: Erna Paris, *Long shadows: Truth, lies and history* (Leader: Jeffrey Dvorkin)

March 6: Scott Weidensaul, *A world on the wing: the global odyssey of migratory birds* (Leader: Sara Shettleworth)

April 3: Thomas Savage, *The power of the dog (1967)* (Leaders: Mary Jane Ashley, Linda Hutcheon)

May 1: Kyle Harper, *From shame to sin: the Christian transformation of sexual morality in late Antiquity* (Leader: David MIlne)

June 5: Steven Johnson, *The ghost map: the story of London's most terrifying epidemic – and how it changed science, cities and the modern world* (Leader: William Logan)

July 3: Niccolo Machiavelli, *The Prince (1532)* (Leader: David Milne)

Aftermath

It has been known since the time of Euclid that it is not possible for a square number to be equal to twice another square number. But squares and twice squares can differ by 1 infinitely often: $3^2 - 2 \times 2^2 = 1$, $7^2 - 2 \times 5^2 = -1$, $17^2 - 2 \times 12^2 = 1$, and so on.

Over the past two millenia, mathematicians have addressed the problem of finding squares that differ from other multiples of squares by 1. In algebraic terms, this means to solve the equation $x^2 - dy^2 = 1$ for integers x and y, where d is a positive integer that is not itself a square. This turns out to be important, since many problems in number theory lead to this equation. If you can find the solution with the smallest positive integers, then a straightforward algorithm will lead you to the complete set of solutions.

The mathematicians of the Indian subcontinent have a long tradition of adeptness in computation, and Brahmagupta (598-668 CE) developed algorithms to get the smallest solutions to equations of this type.

Sometimes finding a solution is child's play. For example $(x, y) = (8, 1)$ satisfies $x^2 - 63y^2 = 1$. Sometimes a little more work is needed: $(x, y) = (31, 4)$ satisfies $x^2 - 60y^2 = 1$. Using the fact that the square of any whole number differs from the product of its two neighbours by 1, we note that $63^2 - 62 \times 64 = 1$. This leads us to the solution $(x, y) = (63, 8)$ for $x^2 - 62y^2 = 1$.

So, there developed a cottage industry of finding solutions to the equation $x^2 - dy^2 = 1$ for various values of the whole number d. You might try it yourself with

values of d less than 20. However, there are some cases where the smallest solution is bigger than might be expected. For example, the smallest solution for $x^2 - 13y^2 = 1$ is $(x, y) = (649, 180)$.

But the real challenge was to find a solution in positive integers to $x^2 - 61y^2 = 1$. In the course of time, Bhaskara the Learned (1114-1185) put his shoulder to the wheel and developed a stepwise approach that in particular revealed that the smallest pair of integers $(x; y)$ that satisfies $x^2 - 61y^2 = 1$ is $(x; y) = (1, 766, 319, 049; 226, 153, 980)$. Quite a *tour de force* when everything had to be done by hand!